EVERY z-LINEAR MAPS IS A FUNCTIONAL p-CONVEX

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ABSTRACT. The main result of the paper is the following: Every z-linear maps is a functional p-convex. We will prove this statement using lemma developed by Kalton and Peck [6] and theorem developed by Aoki and Rolewicz.Based on the definition of functional and using the tool of the triangle inequality prove the following theorem.

1. Introduction

In this paper we show a way that a linear map is a p-convex functional. An important theorem that relates this is was developed by Aoki and Rolewicz is

Theorem 1.1. *Every locally bounded space is p-convex for some $p > 0$. It follows that, if $(X, \|\|)$ is locally bounded, there are positive numbers p and L such that*

$$\left\| \sum_{i=1}^{n} x_i \right\| \leq L \left(\sum_{i=1}^{n} \|x_i\|^p \right)^{1/p} \tag{1.1}$$

for all n and $x_1,\dots,x_n \in X$. In the case of a convex set is linked convex functional concept. The following definition is a z-linear map which will serve to show.

Definition 1.2. *A nonnegative functional p, defined on a real linear space L is called convex, if*

(1) $p(x+y) \leq p(x)+p(y)$.
(2) $p(\alpha x)=\alpha\ p(x)$.

The property (1) is the triangle inequality, we use this to prove that a z-linear map is a p-convex functional.

Definition 1.3. *Let X and Y be two Banach spaces. A map $f{:}X \to Y$ is colled z-linear if it is homogeneous and verifies: $\exists\ C > 0$: $\forall\ x_1,\dots x_n \in X$,*

$$\left\| f\left(\sum_{i=1}^{n} (x_i) \right) - \sum_{i=1}^{n} f\left(x_i\right) \right\| \leq C \left(\sum_{i=1}^{n} \|x_i\| \right) \tag{1.2}$$

equivalently, there exists a constant $M > 0$ such that for each set $x_1,\dots \in X$ such that $\sum_{i=1}^{n} x_1 = 0$ one has

$$\left\| \sum_{i=1}^{n} f\left(x_i\right) \right\| \leq M \sum_{i=1}^{n} \|x_i\| \tag{1.3}$$

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The inequality (1.3) although not clear is another way of enunciating the triangle inequality.

Theorem 1.4. *Every z-linear maps is a functional p-convex*

Proof. Consider a functional φ, and let X,Y two spaces Banach such that define $\varphi : X \to Y$, let vector sequences $x_1,\ldots x_i \in X$, assume $\varphi(x_i) = x_i$ and homogeneous.

Applying triangle inequality two vectors then

$$\|\varphi(x_1) + \varphi(x_2)\| \leq \|\varphi(x_1)\| + \|\varphi(x_2)\| \tag{1.4}$$

for all vectors

$$\left\| \sum_{i=1}^{n} \varphi(x_1) \right\| \leq \sum_{i=1}^{n} \|\varphi(x_i)\| \tag{1.5}$$

by property of inequalities $\exists\ \gamma > 0$ such that

$$\left\| \sum_{i=1}^{n} \varphi(x_1) \right\| \leq \gamma \sum_{i=1}^{n} \|\varphi(x_i)\| \tag{1.6}$$

by Lemma 1.5.[Kalton and Peck] Suppose X and Y are quasi-normed F-spaces. Then exist positive constants r and L such that whenever Y_0 is a dense subspace of Y and $f{:}Y_0 \to X$ is quasi-additive of orden K,

$$\left\| f(\sum_{i=1}^{n} y_i) - \sum_{i=1}^{n} f(y_i) \right\| \leq KL(\sum_{i=1}^{n} \|y_i\|^r)^{1/r} \tag{1.7}$$

for any $y_1,\ldots,y_n \in Y_0$. If $X{=}Y{=}\mathbf{R}$, then L can be taken to equal 1 and r can be taken to equal 1/2. therefore

$$C = KL \tag{1.8}$$

then the theorem is proved, because it meets the definition of z-linear map.

□

Remark 1.5. The argument developed in the show was inspired by the lemmas and theorems described above. It is clear that the constant $M{=}\gamma$ is therefore fulfills the Definition 1.3. The relation[1] that exists between the constants M and C is the following,$C{=}2M$. By Theorem all space is locally bounded p-convex, and one could also analyze the spaces formed z-linear maps, and if those spaces are dense.

2. Appendix A. Some considerations linear maps

Throughout this appendix denote the z-linear maps as f_Z. We know that from the point of view of abstract algebra f_Z is a homomorphism between the two spaces. In the language of category theory is a morphism in the category of vector space on a given field

Given two objects A,B the set of morphisms from A into B denote $Hom_\Delta(A,B)$, where Δ is category. Therefore a f_Z is $Hom_\Delta(A,B)$.

Definition 2.1 (Quasi-Linear Maps)**.** Let X and Y be two quasi-Banach spaces, it is said that a map $\Lambda : X \to Y$ is a quasi-linear map if it is homogeneous and verifies:

$$\exists C > 0 : \forall x, x^* \in X, \|\Lambda(x + x^*) - \Lambda(x) - \Lambda(x^*)\| \leq C(\|x\| + \|x^*\|) \tag{2.1}$$

[1]See [9], Explains the relation between the two constants

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